# A Galerkin Moment Method for the Analysis of an Insulated Antenna in a Dissipative Dielectric Medium

Prodromos E. Atlamazoglou, *Member, IEEE*, and Nikolaos K. Uzunoglu, *Senior Member, IEEE*

***Abstract*— A Galerkin moment method is employed to solve the problem of a dielectric-coated dipole antenna in a dissipative medium. Piecewise sinusoids are used as basis and testing functions. The dielectric coating is modeled by equivalent-volume polarization currents, which are simply related to the conduction current distribution. No additional unknowns are introduced, and the size of the moment-method matrix is the same as that for bare antennas. Exact and approximate formulas for the near electric field are derived. The computed results exhibit excellent agreement with those previously published for a symmetric, as well as an asymmetric insulated dipole. Compared to its existing competitors, the new method appears to be more general and computationally efficient.**

***Index Terms*— Antennas in matter, Galerkin technique, insulated antennas, moment method.**

## I. Introduction

DIELECTRIC-COATED antennas are often preferable over bare ones for use in a conducting medium [1]. The reason is that the often undesirable contact between the antenna and the surrounding space is avoided and, more importantly, the radiation efficiency of the antenna can be improved by insulating all or part of its surface. Coated antennas find use in many diverse areas such as subsurface communications, telemetry, geophysical explorations, and microwave hyperthermia, which is of prime interest to the authors of this paper.

Hyperthermia is the use of heat for the destruction of malignant tissues in the treatment of cancer. The primary objective of any hyperthermia treatment is to raise the temperature within a tumor volume above 42 °C–43 °C for a sufficient period of time, while maintaining the surrounding tissues at temperatures well below 43 °C. The high temperatures inside the tumor can be directly cytotoxic, and have been proven to potentiate the effects of both chemotherapy and radiation therapy.

Insulated antennas are used for localized heating in interstitial or invasive hyperthermia, where dipoles operating at microwave frequencies are inserted into tumors through brachytherapy catheters. Of primary interest in interstitial-applicator modeling is the near field of the antenna, where most of the heating takes place. The near electric field of a dipole in a dissipative medium like a tumor is much more complicated than the far field because of its elliptic polarization.

The near field of a symmetric insulated dipole in a conducting medium was first investigated by King *et al.* [2]. Their results, which were based on an approximate numerical calculation, were improved by Casey *et al.* [3] so that the field in the immediate vicinity of the insulation could be more accurately evaluated. This work was extended by Zhang *et al.* in [4] to include the case of an asymmetric-coated dipole. Clibbon *et al.* [5] developed approximate expressions for this case, and proposed a computationally efficient technique that combines the approximate and exact relations. All of these works are restricted to the case where the complex permittivity of the exterior medium is much greater than that of the insulating layer, which is considered to extend to infinity.

Recently, the finite integration algorithm [6] and the finite-difference time-domain method [7] have been applied for the analysis of interstitial hyperthermia applicators. These approaches are more general and versatile, but they require the discretization of the insulation and part of the surrounding medium, which can be avoided in integral-equation techniques.

One such technique is that proposed in [8]. There, Richmond and Newman used the moment method to solve an integral equation based on the reaction integral by using a piecewise-sinusoidal approximation of the current. That approach bares close resemblance to the one presented in this publication.

However, several differences exist between the two approaches. For example, the method of Richmond and Newman is approximately Galerkin, as they use tubular dipoles for expansion, but filamentary ones for testing. Ours is a true Galerkin method, as both basis and trial functions are associated with tubular dipoles. This choice was made because the true Galerkin solution, although more complicated, is superior as it is characterized by better convergence behavior and variational properties. Furthermore, the publication of Richmond and Newman reported only results such as conductance, susceptance, and resonant length for dielectric-coated antennas in air, while in this paper, the emphasis is on the near electric field radiated by insulated dipoles inside dissipative media.

The technique presented here can be seen as a modification of the piecewise sinusoidal reaction formulation for uninsulated cylindrical antennas. Therefore, Section II starts with the presentation of the bare antenna formulation. Then, the modifications that take into account the dielectric coating are introduced. Exact and approximate expressions for the near

Manuscript received June 23, 1997; revised April 7, 1998.

The authors are with the National Technical University of Athens, Department of Electrical and Computer Engineering, Electroscience Division, Athens, GR-15773, Greece.

Publisher Item Identifier S 0018-9480(98)04950-3.

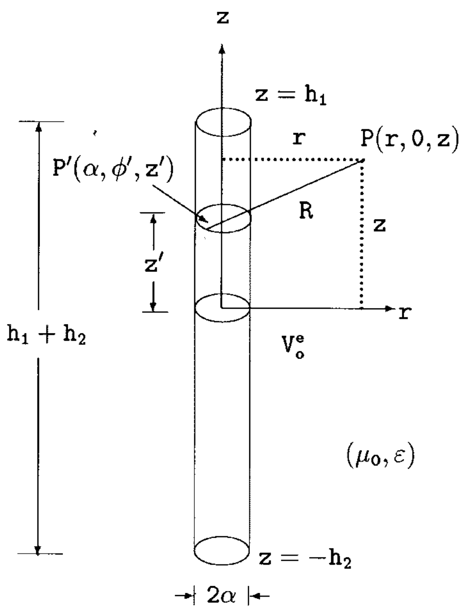


Fig. 1. A cylindrical dipole antenna.

electric field in the ambient medium are derived in Section IV. Sections V and VI contain numerical results and conclusions.

In the following analysis, a time dependence $e^{j\omega t}$ is assumed and suppressed for all sources and fields.

## II. Bare Antenna Formulation

The reaction concept was introduced by Rumsey [9] to simplify the formulation of boundary value problems in electromagnetics. The reaction between two monochromatic sources $a$ and $b$ with the same frequency, consisting of electric currents $\mathbf{J}_a$ and $\mathbf{J}_b$ and magnetic currents $\mathbf{M}_a$ and $\mathbf{M}_b$, is defined by

$$\langle a, b\rangle = \iiint_{V_b} (\mathbf{E}_a \cdot \mathbf{J}_b - \mathbf{H}_a \cdot \mathbf{M}_b)\, dV \tag{1}$$

where the electric and magnetic fields radiated by the sources $a$ and $b$ are $\mathbf{E}_a$, $\mathbf{H}_a$ and $\mathbf{E}_b$, $\mathbf{H}_b$, respectively. The integration volume $V_b$ contains $\mathbf{J}_b$ and $\mathbf{M}_b$.

Fig. 1 shows a bare cylindrical antenna. From the surface equivalence theorem of Schelkunoff, the cylindrical wire can be replaced by the ambient medium if the following current densities are introduced on its surface $S$:

$$\mathbf{J}_S = \hat{n} \times \mathbf{H} \tag{2}$$

$$\mathbf{M}_S = \mathbf{E} \times \hat{n} \tag{3}$$

where $\hat{n}$ is the outward directed unit vector on $S$. By defining $\mathbf{J}_S$, $\mathbf{M}_S$, as in (2) and (3), the total field inside the cylindrical antenna is zero.

An approximate solution to the radiation problem of the bare antenna is obtained by replacing the correct sources $\mathbf{J}_S$ and $\mathbf{M}_S$ with approximate ones, which are adjusted so that their reactions with certain "test" sources are correct. This insures that the approximate sources "look" the same as the correct ones according to the physical tests that are inherent in the problem.

A group of electric test sources $\mathbf{J}_t$ is placed on the surface $S$ of the bare wire. From the reciprocity theorem, we have that the reaction of the sources $\mathbf{J}_S$ and $\mathbf{M}_S$ on the test sources $\mathbf{J}_t$ is equal to the reaction of $\mathbf{J}_t$ on $\mathbf{J}_S$ and $\mathbf{M}_S$

$$\iint_S (\mathbf{E}_t \cdot \mathbf{J}_S - \mathbf{H}_t \cdot \mathbf{M}_S)\, dS = \iint_S \mathbf{E}_S \cdot \mathbf{J}_t\, dS. \tag{4}$$

Below, we consider perfectly conducting antennas and, thus, $\mathbf{M}_S = 0$. The integral (4) can be further simplified by making the well-known thin-wire approximations. These approximations are valid if the antenna radius is much smaller than the ambient medium wavelength, and the antenna length much greater than its radius. According to them, the integrations over the flat-end surfaces of the cylinder and the circumferential component $J_\phi$ of the surface current density are neglected, and the axial component of $J_z$ is considered to be independent of $\phi$. In view of these approximations, the reaction integral equation for a bare antenna reduces to

$$\frac{j\omega\mu_0}{8\pi^2} \int_{-h_2}^{h_1} \int_{-\pi}^{\pi} \left( \int_{-h_2}^{h_1} \int_{-\pi}^{\pi} \left(1 + \frac{1}{k^2}\frac{\partial^2}{\partial z^2}\right) \frac{e^{-jkR}}{R} \cdot I_t(z')\, d\phi'\, dz' \right) I_S(z)\, d\phi\, dz = V_o^e I_t(0) \tag{5}$$

where

$$R = \sqrt{(z-z')^2 + 4\alpha^2 \sin^2\left(\frac{\phi - \phi'}{2}\right)}. \tag{6}$$

Equation (5) is solved via the method of moments [10]. To do this, $I_S(z)$ is expanded in terms of a finite series as follows:

$$I_S(z) = \sum_{n=1}^{N} A_n F_n(z) \tag{7}$$

where $A_n$ are unknown coefficients, and $F_n(z)$ are a known basis set. The piecewise sinusoids are chosen as expansion functions. These are subdomain functions defined by

$$F_n(z) = \begin{cases} \dfrac{\sin(k(z - z_{n-1}))}{\sin(k\Delta z)}, & z_{n-1} \le z \le z_n \\ \dfrac{\sin(k(z_{n+1} - z))}{\sin(k\Delta z)}, & z_n \le z \le z_{n+1} \\ 0, & \text{elsewhere} \end{cases} \tag{8}$$

where $\Delta z = z_n - z_{n-1} = z_{n+1} - z_n$. These functions resemble the natural current distribution on a perfectly conducting thin wire, and allow us to analytically perform two of the integrations in (5) when $k$ is the wavenumber of the surrounding medium. The $F_n(z)$ basis functions are placed in an overlapping array on the surface of the cylindrical antenna so that the current continuity is automatically enforced.

The piecewise sinusoids $F_n(z)$ are also chosen as testing functions. Because the testing sources are identical to the expansion modes in this case, the moment method is an application of Galerkin's technique. By inserting (7) in (5), and enforcing the latter for $N$ distinct test sources $F_n$, the following system of simultaneous linear algebraic equations is obtained:

$$\sum_{n=1}^{N} Z_{mn} A_n = V_m, \qquad m = 1, 2, \cdots, N. \tag{9}$$

Each element $Z_{mn}$ of the coefficient matrix expresses the coupling between the expansion and the testing functions $F_m$ and $F_n$. It is the reaction of the current of $F_n$ to the field radiated by $F_m$. Because of the similarity which characterizes the basis functions selected (they all have the same orientation and equal support), all the elements of every diagonal of $Z$ are identical to each other. This means that $Z$ is a Toeplitz matrix, and only one element from every diagonal needs to be evaluated and stored, resulting in significant savings in computational time and memory. Furthermore, Levinson's method [11] can be used for the fast solution of (9) with only $O(N^2)$ complexity exploiting the symmetric Toeplitz nature of $Z$.

Using (5)–(9) and after some algebra, it is found that

$$Z_{mn} = \frac{j\omega\mu_0}{4\pi k\,\sin^2(k\Delta z)} \int_{z_{m-1}}^{z_{m+1}} \int_{-\pi}^{\pi} \cdot \left( \frac{e^{-jkR_{n+1}}}{R_{n+1}} + \frac{e^{-jkR_{n-1}}}{R_{n-1}} - 2\cos(k\Delta z)\,\frac{e^{-jkR_n}}{R_n} \right) \cdot \sin(k(\Delta z - |z' - z_m|))\,d\phi'\,dz' \tag{10}$$

where

$$R_l = \sqrt{(z'-z_l)^2 + 4\alpha^2\,\sin^2\left(\frac{\phi'}{2}\right)}, \quad l = n+1,\, n-1,\, n. \tag{11}$$

The double integration in (10) must be performed numerically. The integrand in (10) is characterized by singularities at $\phi' = \phi$, $z' = z_{n+1}$, $z_{n-1}$, and $z_n$. These singularities are integrable, and the double numerical integration in (10) can be obtained directly. However, it is preferable to subtract the singularities in order to simplify the calculations and perform the integrations more efficiently. The singularities can be isolated by rewriting the kernels of (10) in the form

$$\int_{-\pi}^{\pi} \frac{e^{-jkR_l}}{R_l}\,d\phi' = \int_{-\pi}^{\pi} \frac{1}{R_l}\,d\phi' - \int_{-\pi}^{\pi} \frac{1-e^{-jkR_l}}{R_l}\,d\phi'. \tag{12}$$

The second integral now has a well-behaved integrand. Furthermore, it may be shown that

$$\int_{-\pi}^{\pi} \frac{1}{R_l}\,d\phi' = \frac{4}{R_l'}\,K\left(\frac{\pi}{2}, p^2\right) \tag{13}$$

where

$$R_l' = \sqrt{(z'-z_l)^2 + 4\alpha^2} \tag{14}$$

$$p^2 = \frac{4\alpha^2}{(z'-z_l)^2 + 4\alpha^2} \tag{15}$$

and $K(\pi/2, p^2)$ is the complete elliptic integral of the first kind with a readily available polynomial approximation algorithm [12] for its computation. The integration with respect to $z'$ in (10) can be performed efficiently with an automatic adaptive algorithm.

## III. Modification for an Insulated Antenna

The formulation of Section II can be extended to take into account the presence of a dielectric coating of an insulated antenna. For the sake of simplicity in the following discussion, it is considered that the insulating layer is homogenous and has the permeability of the surrounding medium.

Using the volume equivalence theorem, the dielectric coating can be replaced with ambient medium and an equivalent-volume polarization current

$$\mathbf{J}_P = j\omega(\varepsilon_2 - \varepsilon)\mathbf{E} \tag{16}$$

where $\mathbf{E}$ is the electric field inside the coating, and $\varepsilon_2$ and $\varepsilon$ denote the permittivities of the insulation and the ambient medium, respectively.

Therefore, the current that models a coated dipole has two components. The first is the surface current $\mathbf{J_S}$, which flows along the $z$-axis on the conductive surface $r = \alpha$ of the antenna. The second is the volume polarization current $\mathbf{J_P}$. This current runs radially inside the region of the insulating layer, and vanishes outside it.

Consequently, for a coated dipole, the reaction integral (4) is modified by replacing $\mathbf{J_S}$ with $\mathbf{J_S} + \mathbf{J_P}$. This means that a volume integral has to be added in the left-hand side of (4). The integration area is the volume of the dielectric coating ($\alpha < r < c$, $-\pi < \phi < \pi$, $-h_2 < z < h_1$), and the integrand is the product of the radial electric field radiated by the test source inside the coating times the polarization current. The right-hand side of (4) remains unchanged, as the axial component of the polarization current is considered negligible on $r = \alpha$:

$$\iint_S E_{tz} J_{Sz}\,dS + \iiint_V E_{tr} J_{Pr}\,dV = \iint_S E_{Sz} J_{tz}\,dS. \tag{17}$$

Ordinarily, $E_{tr}$ and $J_{Pr}$ in the area of the dielectric coating are unknown functions. This means that the introduction of the polarization current in the analysis has, as a result, the increase of the unknowns in the method of moments. However, this is not always so. When the insulating layer is thin, the $E_{tr}$ and $J_{Pr}$ quantities can be expressed approximately as functions of $J_{Sz}$. Therefore, the new unknowns are dependent on the original ones, and this relation keeps the total number of unknowns the same as that of the bare dipole analysis. Naturally, some of the elements of the moment-method matrix are modified. These modifications are determined by the following analysis.

According to the continuity equation, the surface charge density on the conductive surface $r = \alpha$ of the antenna is given by

$$\rho_S = \frac{j}{\omega 2\pi\alpha}\,\frac{dI_S(z)}{dz}. \tag{18}$$

For perfectly conducting cylinders, the $r$ component of the electric flux density $\mathbf{D}$ at their surface is equal to $\rho_S$. The relation $D_r(r=\alpha) = \rho_S$ is also a satisfactory approximation for the case of cylinders with high, but not necessarily, perfect conductivity. As a result, the electric field on the surface $r = \alpha$ of the dipole can be approximated by the expression

$$\mathbf{E}(r=\alpha) = E_r \hat{r} = \frac{D_r}{\varepsilon_2}\,\hat{r} = \frac{\rho_S}{\varepsilon_2}\,\hat{r} = \frac{j}{\omega 2\pi \alpha \varepsilon_2}\,\frac{dI_S(z)}{dz}\,\hat{r}. \tag{19}$$

The $z$ and $\phi$ components of $\mathbf{E}$ inside the dielectric coating are considered negligible. For the $r$ component of $\mathbf{E}$ in the insulation, the following quasi-static approximation can be used:

$$E_r(r,\, z) = \frac{j}{\omega 2\pi r \varepsilon_2}\,\frac{dI_S(z)}{dz}. \tag{20}$$

Substituting the previous expression in the relation for the polarization current (16), we get

$$\mathbf{J}_P = \frac{(\varepsilon\varepsilon_2)}{2\pi\varepsilon_2 r}\,\frac{dI_S(z)}{dz}. \tag{21}$$

From the above analysis, it is obvious that each basis function used for the expansion of the conduction current is related to a part of the radial volume polarization current. Therefore, each element of the moment-method matrix $Z_{mn}$ for an insulated antenna has an additional term. Using (17) and (21), it is found that this extra term is given by

$$\begin{aligned}\Delta Z_{mn} &= \int_{\alpha}^{c}\int_{-h_2}^{h_1}\int_{-\pi}^{\pi} E_{\mathrm{trm}}(r',\, z') J_{Prn}(z') r'\, d\phi'\, dz'\, dr' \\ &= \int_{\alpha}^{c}\int_{z_{n-1}}^{z_{n+1}} \frac{(\varepsilon-\varepsilon_2)}{\varepsilon_2}\, E_{\mathrm{trm}}(r',\, z')\,\frac{dF_n(z')}{dz'}\, dz'\, dr'.\end{aligned} \tag{22}$$

The integration with respect to $\phi'$ is performed with the assumption that $E_{tr}$ and $J_{\mathrm{Pr}}$ are independent of $\phi'$. The limits $\alpha$ and $c$ of the integration, with respect to $r'$, are the inner and outer radii of the insulation.

Again making use of a quasi-static approximation, the testing source field $E_{\mathrm{trm}}$ in (22) can be expressed as a function of the derivative with respect to $z'$ of the conduction current's expansion function $F_m : z'$):

$$E_{\mathrm{trm}}(r',\, z') = \frac{j}{\omega 2\pi r\varepsilon}\,\frac{dF_m(z')}{dz'}. \tag{23}$$

Inserting (23) in (22), $\Delta Z_{mn}$ becomes

$$\begin{aligned}\Delta Z_{mn} &= \int_{\alpha}^{c}\int_{z_{n-1}}^{z_{n+1}} \frac{(\varepsilon-\varepsilon_2)}{\varepsilon_2}\,\frac{j}{\omega 2\pi r\varepsilon}\,\frac{dF_m(z')}{dz'}\,\frac{dF_n(z')}{dz'}\, dz'\, dr' \\ &= \frac{j}{2\pi\omega\varepsilon}\int_{\alpha}^{c}\frac{(\varepsilon-\varepsilon_2)}{\varepsilon_2 r'}\, dr' \int_{z_{n-1}}^{z_{n+1}} \frac{dF_m(z')}{dz'}\,\frac{dF_n(z')}{dz'}\, dz' \\ &= \frac{j}{2\pi\omega\varepsilon}\,\frac{(\varepsilon-\varepsilon_2)}{\varepsilon_2}\,\ln\left(\frac{c}{\alpha}\right)\int_{z_{n-1}}^{z_{n+1}} \frac{dF_m(z')}{dz'}\,\frac{dF_n(z')}{dz'}\, dz'.\end{aligned} \tag{24}$$

The elements of $\Delta Z$ can be evaluated analytically if piecewise sinusoids are used as basis and testing functions. This means that the computational cost required for the construction of the moment-method matrix for an insulated antenna is almost equal to that for a bare antenna, as no numerical integrations are needed for the evaluation of $\Delta Z$.

The $\Delta Z$ matrix is tridiagonal in that it has nonzero elements only on the diagonal (plus or minus one column). This is a result of the form of the piecewise sinusoid basis functions. Furthermore, just like $Z$, $\Delta Z$ is symmetric and Toeplitz. It can easily be found that its elements are given by the relations

$$\begin{aligned}\Delta Z_{n,\,n} &= \frac{jk}{2\pi\omega}\,\frac{(\varepsilon-\varepsilon_2)}{\varepsilon\varepsilon_2}\,\ln\left(\frac{c}{\alpha}\right) \\ &\quad\cdot\left(\frac{k\Delta z}{\sin^2(k\Delta z)} + \frac{1}{\tan(k\Delta z)}\right)\end{aligned} \tag{25}$$

$$\begin{aligned}\Delta Z_{n,\,n-1} &= \frac{-jk}{4\pi\omega\sin(k\Delta z)}\,\frac{(\varepsilon-\varepsilon_2)}{\varepsilon\varepsilon_2}\,\ln\left(\frac{c}{\alpha}\right) \\ &\quad\cdot\left(1 + \frac{k\Delta z}{\tan(k\Delta z)}\right)\end{aligned} \tag{26}$$

$$\Delta Z_{n-1,\,n} = \Delta Z_{n,\,n-1}. \tag{27}$$

According to the preceding analysis, the implementation of the moment method for a dielectric-coated dipole has, as a result, the following $N \times N$ linear system:

$$\sum_{n=1}^{N}(Z_{mn}+\Delta Z_{mn})A_n = V_m, \qquad m = 1,\, 2,\, \cdots,\, N \tag{28}$$

where $A$, $Z$, and $V$ are the matrices that appear in the bare antenna formulation, while the elements of $\Delta Z$ express the effect of the insulation in the analysis.

In case that a portion of the antenna is not coated, then the part of $\Delta Z$ that results from integration along that portion is zero. This means that the modeling of the insulation by equivalent-volume polarization currents is general enough to treat partly coated and partly bare dipoles. This generalization is simple, does not introduce additional numerical cost, and preserves the symmetry of the moment-method matrix.

Although in the beginning of this section the assumption was made that the dielectric coating is homogenous, this is not necessary in practice. Expression (22) is sufficiently general to describe the effect of an inhomogenous coating when its permittivity is a function of $r$ and $z$ only. If the insulation's permittivity depends on $\phi$, then the analysis becomes extremely complicated, as $\mathbf{J}_S$ and $\mathbf{J_P}$ will also be functions of $\phi$. This case is not examined here.

When $\varepsilon_2$ is a function of $r$ only, then the sole modification required is in the evaluation of the integral with respect to $r'$ in (24). If the coating's permittivity is characterized by a sufficiently simple radial dependence, then the integral can still be obtained in closed form. In case the coating is multilayered, and the $l$ layer extends from $r = r_l$ to $r_{l+1}$ and has constant permittivity $\varepsilon_l$, then the expression that gives $\Delta Z$ takes the form

$$\begin{aligned}\Delta Z_{mn} = &\frac{j}{2\pi\omega}\sum_{l=1}^{L}\frac{(\varepsilon-\varepsilon_2)}{\varepsilon\varepsilon_2}\,\ln\left(\frac{r_{l+1}}{r_l}\right) \\ &\cdot\int_{z_{n-1}}^{z_{n+1}}\frac{dF_m(z')}{dz'}\,\frac{dF_n(z')}{dz'}\, dz'.\end{aligned} \tag{29}$$

The above relation can be significantly simplified, with the introduction of an equivalent homogenous coating that replaces the $L$ layers of the real coating. This equivalent insulation extends from $r = r_1 (= \alpha)$ to $r_{L+1} (= c)$ and is characterized by a dielectric constant $\varepsilon_{eL}$ given by

$$\varepsilon_{eL} = \varepsilon_1 \frac{\ln\left(\dfrac{r_{L+1}}{r_1}\right)}{\displaystyle\sum_{l=1}^{L} \frac{\varepsilon_1}{\varepsilon_l} \ln\left(\frac{r_{l+1}}{r_l}\right)}. \tag{30}$$

## IV. The Electric Field in the Surrounding Medium

Due to rotational symmetry, the electric field radiated by the antenna in the ambient medium has only $z$ and $r$ components, which are independent of $\phi$.

The field at a point $(r, z)$ is given by the superposition of the partial fields radiated by the expansion function currents with weights at the coefficients determined via the moment method as follows:

$$\mathbf{E}(r, z) = \sum_{n=1}^{N} A_n \mathbf{E}_n(r, z) \tag{31}$$

where $\mathbf{E}_n(r, z)$ is the electric field at $(r, z)$ of an imaginary source with the characteristics (length, orientation, current distribution) of the piecewise sinusoid $n$.

The $\mathbf{E}_n(r, z)$ field can be expressed in terms of the vector potential $\mathbf{A_n}$ as

$$\mathbf{E}_n = \frac{1}{j\omega\mu_0\varepsilon} \nabla(\nabla \cdot \mathbf{A}_n) - j\omega\mathbf{A}_n. \tag{32}$$

Because of the thin-wire approximations, the vector potential $\mathbf{A}_n$ has only a $z$-component, which is given by

$$A_{zn}(r, z) = \frac{\mu_0}{4\pi} \int_{z_{n-1}}^{z_{n+1}} K(z - z') F_n(z')\, dz' \tag{33}$$

where

$$K(z - z') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{-jkR}}{R}\, d\phi' \tag{34}$$

represents the cylindrical antenna kernel and

$$R = \sqrt{(z - z')^2 + r^2 + a^2 - 2ra\, \cos(\phi')}. \tag{35}$$

The expressions that give $E_{zn}$ and $E_{rn}$ as functions of $A_{zn}$ are

$$E_{zn}(r, z) = \frac{1}{j\omega\mu_0\varepsilon} \frac{\partial^2}{\partial z^2} A_{zn}(r, z) - j\omega A_{zn}(r, z) \tag{36}$$

$$E_{rn}(r, z) = \frac{1}{j\omega\mu_0\varepsilon} \frac{\partial^2}{\partial r \partial z} A_{zn}(r, z). \tag{37}$$

Substituting (33) into (36) yields

$$\begin{aligned} E_{zn}(r, z) &= -\frac{jk}{8\pi^2\omega\varepsilon\, \sin(k\Delta z)} \int_{-\pi}^{\pi} \\ &\quad \cdot \left( \frac{e^{-jkR_{n+1}}}{R_{n+1}} + \frac{e^{-jkR_{n-1}}}{R_{n-1}} - 2\, \cos(k\Delta z) \frac{e^{-jkR_n}}{R_n} \right) d\phi' \end{aligned} \tag{38}$$

where

$$R_l = \sqrt{(z - z_l)^2 + r^2 + \alpha^2 - 2r\alpha\, \cos(\phi')}, \quad l = n+1,\, n-1,\, n. \tag{39}$$

This formula shows that $E_{zn}$ is the superposition of spherical waves which originate at $z_{n+1}$, $z_{n-1}$, and $z_n$. These are points of reflection and the locations of maximum positive and negative charges for the imaginary dipole associated with the piecewise sinusoid $n$.

By inserting (33) into (37), the $r$ component of the $n$ expansion function's electric field is found to be

$$\begin{aligned} E_{rn}(r, z) &= \frac{jk}{8\pi^2\omega\varepsilon\, \sin(k\Delta z)} \int_{-\pi}^{\pi} \frac{r - \alpha\, \cos(\phi')}{r^2 + \alpha^2 - 2r\alpha\, \cos(\phi')} \\ &\quad \cdot \left( \frac{e^{-jkR_{n+1}}}{R_{n+1}} (z - z_{n+1}) + \frac{e^{-jkR_{n-1}}}{R_{n-1}} (z - z_{n-1}) \right. \\ &\quad \left. - 2\, \cos(k\Delta z) \frac{e^{-jkR_n}}{R_n} \right) (z - z_n)\, d\phi'. \end{aligned} \tag{40}$$

An adaptive quadrature routine based on the Gauss Kronrod rules, is used to perform the numerical integrations in (38) and (40). The integrand of expression (38) is highly peaked at, or in, the vicinity of $\phi = \phi'$, $z = z_{n+1}$, $z_{n-1}$, $z_n$, and $r$ close to $c$. For the axial-field computation at these points, it is a good idea to first remove the singular terms from (38), and then perform the numerical integration in the remaining well-behaved integrands. The singular terms can be transformed to complete elliptic integrals of the first kind, for which polynomial approximations exist, as was shown in Section II.

For points not close to the surface of the insulation, the following approximate versions of (38) and (40) can be used:

$$\begin{aligned} E_{zn}(r, z) &\doteq -\frac{jk}{4\pi\omega\varepsilon\, \sin(k\Delta z)} \\ &\quad \cdot \left( \frac{e^{-jkR'_{n+1}}}{R'_{n+1}} + \frac{e^{-jkR'_{n-1}}}{R'_{n-1}} - 2\, \cos(k\Delta z) \frac{e^{-jkR'_n}}{R'_n} \right) \end{aligned} \tag{41}$$

$$\begin{aligned} E_{rn}(r, z) &\doteq \frac{jk}{4\pi\omega\varepsilon r\, \sin(k\Delta z)} \\ &\quad \cdot \left( \frac{e^{-jkR'_{n+1}}}{R'_{n+1}} (z - z_{n+1}) + \frac{e^{-jkR'_{n-1}}}{R'_{n-1}} (z - z_{n-1}) \right. \\ &\quad \left. - 2\, \cos(k\Delta z) \frac{e^{-jkR'_n}}{R'_n} (z - z_n) \right) \end{aligned} \tag{42}$$

where

$$R'_l = \sqrt{(z - z_l)^2 + r^2}, \qquad l = n+1,\, n-1,\, n. \tag{43}$$

The results obtained with (42) show excellent agreement with those from (40) for almost all values of $r$. However, the use of (41) led to accurate values only in the relatively far region ($R'_l \geq 4c$). The reason for this must be the more complicated radial dependence of the electric field's axial component.

An interesting and promising idea is the combination of both (38) and (41) in the computation of the weighted sum (31). The use of the more accurate, but computationally expensive (38), can be limited at the terms for which $R_l' \leq 4c$. The number of these depends on the total number of basis functions used and the dimensions of the antenna, but usually it will be small (three or four terms at most). At the remaining terms, the numerically approximate (41) can be safely chosen.

The electric field radiated in the ambient medium by the volume polarization current is neglected, as it is almost two orders of magnitude smaller than the conduction current's contribution. This is due to the higher powers of $R$, which appear on the denominators of the expressions that describe the polarization current's field. These expressions turned out to be too complicated and, thus, their evaluation is not worth being performed, as they lead to extremely small and negligible contributions.

## V. Numerical Results and Validation

In order to investigate the accuracy and efficiency of the proposed method, a symmetric insulated antenna (analyzed in [2] and [3]) is modeled with it. It is a half-wave dipole inside an air-filled plastic tube. The radius of the inner conductor is $\alpha = 0.47$ mm. The half-length of the dipole is $h = 3.1$ cm, and it is operated at $f = 915$ MHz with $V_0^e = 1$ V. Its dielectric coating has two layers: an inner layer of air with outer radius $b = 0.584$ mm and relative permittivity $\varepsilon_{2r} = 1$, and an outer layer (plastic tube) with $c = 0.80$ mm and $\varepsilon_{3r} = 1.78$, respectively. These two layers can be substituted by an equivalent one, with relative permittivity $\varepsilon_{2er} = 1.373$. The ambient medium has the electrical properties of human brain tissue. Its real relative permittivity is $\varepsilon_{4r} = 42.5$, and it is also characterized by a conductivity $\sigma_4 = 0.88$ S/m. The complex wavenumber of this medium is $k_4 = 127.5 - j25m^{-1}$.

The thin-wire approximations are valid for the above antenna, as its radius is two orders of magnitude smaller than the wavelength, while the radius to half-length ratio is $(\alpha/h) = 0.015$. It is obvious that the antenna is electrically thin. Furthermore, the thickness of the insulating shell is very small ($c - a = 0.33$ mm). Consequently, the polarization current that runs through it can be considered a dependent unknown.

As was noticed in Section IV, numerical integration with respect to $\phi'$ is only required for the computation of the field values close to the antenna. For this integration, an adaptive quadrature is utilized for better efficiency. The results of Casey *et al.* were extracted using [3, eqs. (11a) and (11b)]. For the evaluation of these expressions, the same numerical integration routines with the moment method were used for comparison purposes.

In Fig. 2, the magnitude of the complex current distribution on the surface of the dipole is drawn for various numbers of basis functions. In the same figure, the current distribution obtained from King's theory [2] is included. This distribution has a sinusoidal form with a complex wavenumber different from that of the surrounding medium. It is obvious from Fig. 2

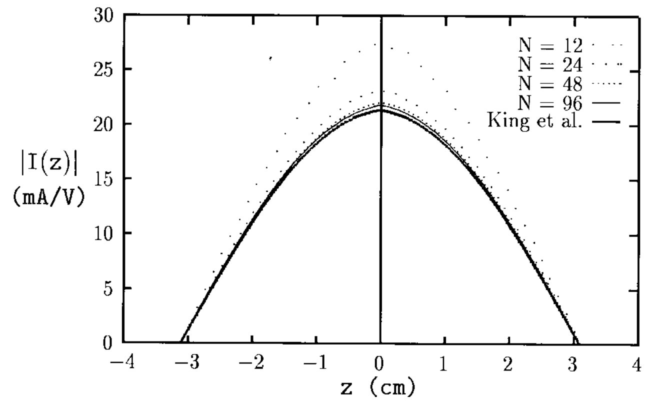


Fig. 2. Current distribution of a symmetric insulated dipole in human brain tissue ($\alpha = 0.47$ mm, $b = 0.584$ mm, $c = 0.8$ mm, $h_1 = h_2 = 3.1$ cm, $f = 915$ MHz, $\varepsilon_{2r} = 1$, $\varepsilon_{3r} = 1.78$, $\varepsilon_{4r} = 42.5$, $\sigma_4 = 0.88$ S/m, $k_4 = 127.5 - j25m^{-1}$).

TABLE I
Input Impedance $Z_S$ for the Symmetric Insulated Dipole Described in Fig. 2

| N | $R_S + jX_S$ |
|---|---|
| 12 | 32.201 - j16.618 |
| 24 | 41.562 - j11.915 |
| 48 | 44.093 - j10.914 |
| 96 | 44.660 - j10.550 |
| King et al. | 45.638 - j10.715 |

that as the number of basis functions increases, the $|I(z)|$ distributions obtained with the moment method successfully approximate the true current distribution, as King's results have been experimentally verified. Additionally, it is noticed that the moment method exhibits fast convergence and, as a result, the extraction of satisfactory results can be achieved with relatively few basis functions per wavelength. This means that the moment method for this particular problem is not only accurate, but also computationally efficient.

Table I contains the values of the dipole's input impedance computed for various discretizations along with the impedance provided by King's theory. It is evident that the moment-method values begin to stabilize, and compare favorably to each other and to King's approximation, once 48 or more basis functions are used.

Fig. 3 displays the magnitude of the axial electric field $|E_z|$ as a function of the radial distance from the dipole's axis with $z$ constant ($z = 5$ mm) and with the number of basis functions $N$ as parameter. In the same figure, the corresponding $|E_z|$ distribution determined with the use of expressions derived by Casey *et al.* [3] is superimposed. As is expected from the $|I_z|$ results, the $|E_z|$ curves also converge quickly as $N$ increases, and their agreement with the field values from Casey *et al.* soon becomes excellent. Even close to (almost on) the surface of the antenna where the determination of the field is very difficult due to its elliptical polarization, the correspondence remains satisfactory. As Casey *et al.* pointed out, $|E_z|$ for $r > c$ decays rapidly as $r$ grows.

Fig. 4 compares our results with those of Casey *et al.* for the radial component of the electric field at $z = 5$ mm in the surrounding space. Just like the axial component case, the moment-method radial-field distributions soon converge

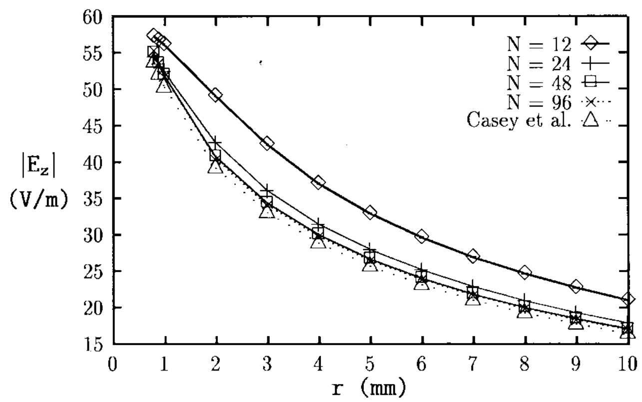


Fig. 3. Magnitude of the axial electric field in the surrounding medium for the insulated dipole described in Fig. 2.

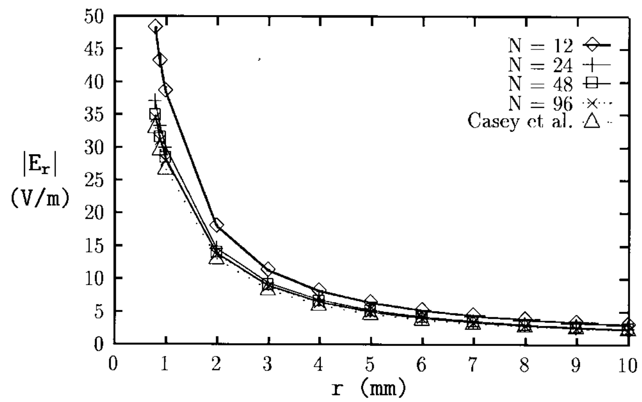


Fig. 4. Magnitude of the radial electric field in the ambient medium for the antenna described in Fig. 2.

and exhibit high accordance with the Casey *et al.* curve. The $E_r$ component is characterized by simpler radial dependence, compared to $E_z$, since it decreases as $1/r$, away from the conducting surface of the dipole. Unlike the radial component, $E_z$ increases away from the conducting surface inside the dielectric insulation, and then monotonically decreases in the dissipative ambient medium.

The field computations with (38) and (40) are significantly faster than the use of (11a) and (11b) if the same algorithm is used for the $\phi'$ numerical integration. The reason for this is that instead of the double integration with respect to $\phi'$ and $z'$ in the expressions of Casey *et al.*, only a sum of one-dimensional numerical integrations with respect to $\phi'$ is required in evaluating (38) and (40). As the moment method converges fast, the number of basis functions needed for sufficient accuracy can be kept relatively small and, consequently, the weighted sum of $E_{zn}$ or $E_{rn}$ is computationally cheaper than the $z'$ integration of the formulas by Casey *et al.* For example, the moment-method results shown in Figs. 3 and 4 for $N = 48$ were obtained almost three times faster than the field values at the same points calculated with the double integrations of Casey *et al.*

Of course, in the moment-method approach, the elements of matrix $Z$ have to be computed in order to determine the initially unknown coefficients of the basis functions. However, these calculations can be performed very efficiently by exploiting the symmetric Toeplitz nature of the $Z$-matrix and extracting the singularities from the double integral of (10). In practice, the computational cost of the coefficient determination is roughly equal to that of finding the field at two or three points extremely close to the outer surface of the dipole's coating using the expressions of Casey *et al.* This relatively low cost has to be paid only once and, afterwards, the coefficients can be used for the field computations in large quantities of positions around the antenna. Obviously, it is a price well worth being paid, as it leads to substantial computational savings during the field calculations.

In the approach of King *et al.*, the $\phi'$ integrations in [3, eqs. (11a) and (11b)] were approximated, resulting in single-variable integral expressions for the electric-field components. As is stated in [2] and [3], these approximations place an additional restriction on the thickness of the insulating layer, and reduce the accuracy of the field evaluated near the insulation's surface. However, for the radial component, and even for the axial at radial distances $r \geq 4c$, the approximate expressions provide satisfactory accuracy very efficiently. As was shown in Section IV, similar approximations can be introduced to (38) and (40), transforming them into (41) and (42), respectively. These approximate expressions have the $\phi'$ numerical integration removed from them, and they require only the computation of a weighted sum. Furthermore, (38) and (40) can be combined with (41) and (42) in the same weighted sum, resulting in both an accurate and numerically cheap form, which competes successfully with the combination expressions provided by Clibbon *et al.* [5].

Next, the moment method is used to study the asymmetric model for interstitial antennas proposed by Zhang *et al.* in [4]. With this model, an interstitial applicator is represented as an asymmetric dipole having two arms of different length. The antenna we simulate has gap-to-tip length $h_1 = 2$ cm, while in [4], it is found that $h_2 = 5h_1 = 10$ cm. The inner conducting cylinder has a diameter of 0.95 mm, and is coated with two layers. The first is a layer of air ($\varepsilon_{2r} = 1$) and has an outer diameter of 1.168 mm. The second layer has relative permittivity $\varepsilon_{3r} = 3.5$, and an outer diameter of 1.6 mm. A homogenous equivalent coating that can substitute for these two layers must have relative electric permittivity $\varepsilon_{2e} = 1.758$. The operating frequency of the dipole is again 915 MHz, and $V_0^e$ is considered to be 1 V. The electric properties of the surrounding medium are those of muscle tissue ($\varepsilon_{4r} = 51.0$ and $\sigma_4 = 1.28$ S/m). The wavenumber in this space is $k_4 = 140.7 - j32.8m^{-1}$.

The magnitude of the asymmetric dipole's current distribution, as it is approximated by the moment method for different numbers of basis functions, is shown in Fig. 5 along with the corresponding distribution provided by the theory of Zhang *et al.* [4]. The moment-method results display convergence, which is more evident for big values of $N$. The discrepancies between the curves of our approach and that of Zhang *et al.*, which exist for very coarse discretizations ($N = 12, 24$), are minimized for $N \geq 48$ and become negligible. The values of the antenna's input impedance obtained by the moment method for various $N$ together with the impedance estimated by Zhang's theory are presented in Table II. It is clear that the input impedance approximations are characterized by a

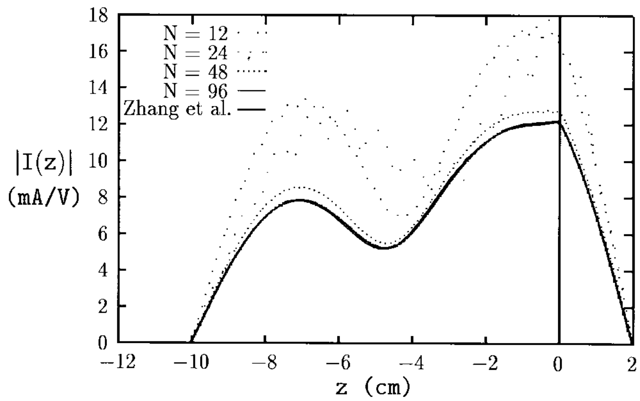


Fig. 5. Current distribution of an asymmetric-coated antenna in muscle tissue ($\alpha = 0.47$ mm, $b = 0.584$ mm, $c = 0.8$ mm, $h_1 = 2$ cm, $h_2 = 10$ cm, $f = 915$ MHz, $\varepsilon_{2r} = 1$, $\varepsilon_{3r} = 3.5$, $\varepsilon_{4r} = 51.0$, $\sigma_4 = 1.28$ S/m, $k_4 = 140.7 - j32.8m^{-1}$).

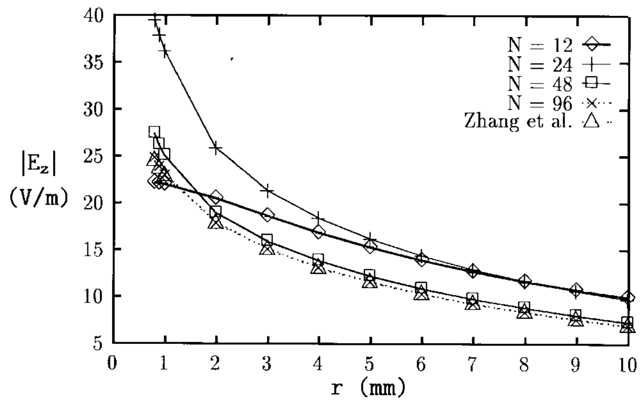


Fig. 6. Magnitude of the axial electric field in the ambient medium for the asymmetric insulated antenna described in Fig. 5.

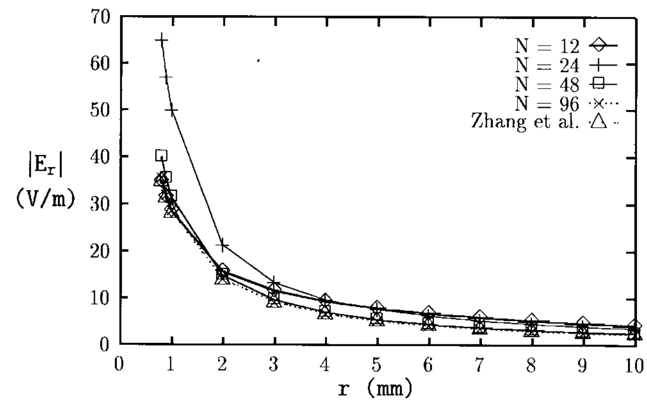


Fig. 7. Magnitude of the radial electric field in the ambient medium for the coated dipole described in Fig. 5.

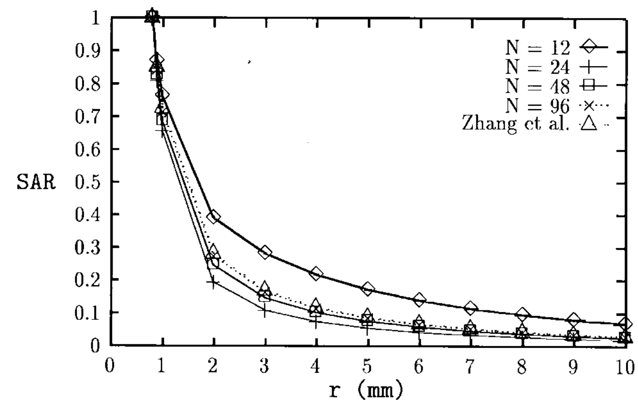


Fig. 8. Normalized SAR distribution at $z = 5$ mm versus the radial distance from the axis of the antenna described in Fig. 5.

TABLE II
INPUT IMPEDANCE $Z_S$ FOR THE ASYMMETRIC INSULATED DIPOLE DESCRIBED IN FIG. 5

| N | $R_S + jX_S$ |
|---|---|
| 12 | -35.506 - j48.379 |
| 24 | 57.299 - j20.807 |
| 48 | 73.500 - j29.378 |
| 96 | 75.963 - j32.437 |
| Zhang et al. | 74.858 - j33.643 |

convergence behavior similar to that observed for the current distribution.

In Figs. 6 and 7, the moment-method distributions for the magnitude of the axial and the radial electric field along the $r$-axis at $z = 5$ mm for various $N$ are compared with the respective distributions computed with the expressions of Zhang's publication. Both figures show that the moment-method field results converge easily to the verified values offered by Zhang's theory.

A quantity of primary interest in hyperthermia is the specific absorption rate (SAR), which is closely related to the rate of temperature increase in a biological medium. SAR is defined as the spatial distribution of energy absorbed per unit mass and is measured in watts/kilograms. The SAR is a function of the electric field radiated by an interstitial antenna, and this relation is expressed in the form

$$\mathrm{SAR} = \frac{1}{2}\,\frac{\sigma}{\rho}\,|E|^2 = \frac{\sigma}{2\rho}\,(|E_z|^2 + |E_r|^2) \tag{44}$$

where $\sigma$ is the conductivity of the ambient medium (siemens/meters), $\rho$ is its density (kilogram/meter$^3$), and $|E|$ the magnitude of the electric field (volts/meters).

Fig. 8 displays the SAR distributions determined with the moment method along the radial axis for $z = 5$ mm. In this figure, the displayed SAR values are normalized to their maximum values, as the true value of the gap generator's potential $V_0^e$ is unknown. These curves clearly reveal the rapid decrease of SAR with radial distance due to the high conductivity of the ambient medium. This is the reason interstitial antennas in hyperthermia practice are used in arrays of four or more dipoles positioned around the treated tumor. The analysis of such arrays with the moment method will be investigated in a future paper.

## VI. CONCLUSIONS

In this paper, a Galerkin moment method for the determination of the current distribution of a coated antenna in a dissipative dielectric medium is presented. The dielectric coating is modeled by equivalent-volume polarization currents in a way that prevents the introduction of additional unknowns and numerical volume integrations. The current distribution obtained is used to calculate the input impedance of the antenna and the radiated field in the ambient medium. Comparisons of the computed results, with values provided by previously published methods, show good agreement for symmetric as well as asymmetric models of interstitial hyperthermia applicators. For the field computations, the moment method appears to be more computationally efficient than the use of King's model for insulated antennas.

An advantage of the King's model is the insight it provides in the operation of a coated dipole as it simulates it with a transmission line. That model leads to an approximate, but closed-form, expression of the dipole's current distribution. However, that expression is not general, but restricted to the case where the complex permittivity of the ambient medium is much greater than that of the dielectric coating. The moment method we propose is not subject to that restriction, thus, it has a wider range of applicability. Furthermore, the transmission-line model considers that the insulating shell in which the dipole is embedded extends to infinity while, in our approach, the dielectric coating terminates at the ends of the inner conductor. Thus, the moment-method model is closer to the physical situation. Additionally, the insulation modeling by equivalent polarization currents is general enough to allow the treatment of inhomogenous coatings or even partially coated antennas, which present very serious (even insurmountable) difficulties to King's model.

However, the most important advantage of the method presented over its predecessors is that it can be easily extended to apply in more complicated cases. By incorporating the modifications that account for the coating to more advanced and general solvers, insulated antennas with more complex geometries (e.g., curved dipoles) can be successfully modeled. A promising option appears to be the coupling of piecewise-sinusoidal reaction formulation to a volume integral equation method for the treatment of inhomogeneities in the surrounding space.

## References


[1] R. W. P. King and G. S. Smith, *Antennas in Matter.* Cambridge, MA: MIT Press, 1981, p. 6.

[2] R. W. P. King, B. S. Trembly, and J. W. Strohbhen, "The electromagnetic field of an insulated antenna in a conducting or dielectric medium," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-31, pp. 574–583, July 1983.

[3] J. P. Casey, and R. Bansal, "The near field of an insulated dipole in a dissipative dielectric medium," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-34, pp. 459–463, Apr. 1986.

[4] Y. Zhang, N. V. Dubal, R. Takemoto-Hambleton, and W. T. Joines, "The determination of the electromagnetic field and SAR pattern of an interstitial applicator in a dissipative dielectric medium," *IEEE Trans. Microwave Theory Tech.,* vol. 36, pp. 1438–1443, Oct. 1988.

[5] K. L. Clibbon, and A. McCowen, "Efficient computation of SAR distributions from interstitial microwave antenna arrays," *IEEE Trans. Microwave Theory Tech.,* vol. 42, pp. 595–600, Apr. 1994.

[6] G. Schaller, J. Erb, and R. Engelbrecht, "Field simulation of dipole antennas for interstitial microwave hyperthermia," *IEEE Trans. Microwave Theory Tech.,* vol. 44, pp. 887–895, June 1996.

[7] J.-C. Camart, D. Despretz, M. Chivé, and J. Pribetich, "Modeling of various kinds of applicators used for microwave hyperthermia based on the FDTD method," *IEEE Trans. Microwave Theory Tech.,* vol. 44, pp. 1811–1818, Oct. 1996.

[8] J. H. Richmond, and E. H. Newman, "Dielectric coated wire antennas," *Radio Sci.,* vol. 11, no. 1, pp. 13–20, Jan. 1976.

[9] V. H. Rumsey, "Reaction concept in electromagnetic theory," *Phys. Rev.,* vol. 94, no. 6, pp. 1483–1491, June 1954.

[10] R. F. Harrington, *Field Computation by Moment Methods.* New York: Macmillan, 1968, pp. 68–75.

[11] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes: The Art of Scientific Computing.* Cambridge, U.K.: Cambridge Univ. Press, 1989, pp. 47–52.

[12] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions.* New York: Dover, 1970, pp. 589–591.


**Prodromos E. Atlamazoglou** (S'93–M'95) was born in Athens, Greece, in 1969. He received the Diploma degree in electrical engineering and the Ph.D. degree from the National Technical University of Athens (NTUA), Athens, Greece, in 1991 and 1997, respectively.

From 1993 to 1996, he worked as System Manager at the Athens High-Performance Computing Laboratory. His research interests focus on finite-element modeling, integral equations, multigrid and domain decomposition methods, and on parallel programming.

Dr. Atlamazoglou is a member of the Technical Chamber of Greece, and is currently performing his national service.

**Nikolaos K. Uzunoglu** (M'82–SM'97) was born in Constantinople, Turkey, in 1951. He received the B.Sc. degree in electronics from the Technical University of Instabul, Instabul, Turkey, in 1973, and the M.Sc. and Ph.D. degrees from the University of Essex, Essex, U.K., in 1974 and 1976, respectively.

From 1977 to 1984, he worked as a Research Scientist at the Office of Research and Technology of the Hellenic Navy. In 1984, he was elected Associate Professor in the Department of Electrical Engineering, National Technical University of Athens, Athens, Greece, and, in 1987, became a full Professor. In 1986, he was elected Vice Chairman of the Department of Electrical Engineering, and, in 1988, was elected Chairman of the same Department. He was twice reelected as Chairman in 1990 and 1992. In 1991, he was elected and appointed Director of the Institute of Communication and Computer Systems, an independent research establishment associated with the National Technical University of Athens. His research interests include electromagnetic scattering, propagation of electromagnetic scattering waves, fiber-optics telecommunications, and high-speed circuits operating at gigabit/second rates. He has authored or co-authored 100 publications in refereed international journals, and has published three books in Greek on microwaves, fiber-optics telecommunications, and radar systems. Since 1988, he has been the national representative of Greece to COST, Technical Telecommunications Committee, actively participating in several COST telecommunications projects. Further, he has been project manager of several RACE, ESPRIT, and National Research and Development Projects in the fields of telecommunications and biomedical engineering applications.

Dr. Uzunoglu received the International G. Marconi Award in Telecommunications in 1981.